Magnetars

Robert C. Duncan* and Christopher Thompson†

*Dept. of Astronomy, University of Texas, Austin, TX 78712
†CITA, 60 St. George Street, Toronto, Ontario M5S 1A1, Canada

Magnetars are defined as neutron stars with dipole fields in excess of $B_{QED} \equiv m_e^2 c^3 / e\hbar = 4.4 \times 10^{13}$ G. We describe how such stars could form through the action of an $\alpha-\Omega$ dynamo during the first 10-30 seconds after neutron star birth. Magnetars constitute a class of neutron stars distinct from pulsars, in which magnetic energy, rather than rotational energy, plays the dominant role in powering emissions. Possible observable manifestations of magnetars include soft gamma repeaters (SGRs), anomalous X-ray pulsars (AXPs), and classical gamma-ray bursts (GRBs). AXPs are soft-spectrum, pulsating X-ray sources with histories of uniform spindown and no evident companions. We estimate their recoil velocities and, in the context of the magnetar model, their dipole magnetic field strengths.

I. NEUTRON STAR DYNAMOS AND THE MAGNETAR CONJECTURE

Neutron stars are born in the intense heat of gravitational collapse. Most of the energy that is released, $GM^2/R \sim 3 \times 10^{53}$ ergs, is lost to neutrinos, which diffuse out of the stellar interior during the first $\sim 10-30$ s. Numerical simulations of hot, young neutron stars show vigorous mixing due to transient Rayleigh-Taylor-like instabilities in the first second after core bounce (1,2) with convection continuing through much of the neutrino diffusion epoch (3). Convection beneath the neutrinosphere (the surface at which the star becomes transparent to neutrinos) can be driven by steady neutrino diffusion within a hot, hydrostatic nuclear fluid, for reasons explained in the Appendix.

Neutron star convection is turbulent (3,4), with an eddy overturn time

$$\tau_{con} \sim 3 \ L_{\nu,52}^{-1/3} \ \rho_{14}^{1/3} \ \left( \frac{f_{con}}{0.1} \right)^{-1/3} \ \left( \frac{\ell_p}{1.5 \text{ km}} \right) \ \text{ms}$$

in mixing-length theory. Here, $L_{\nu,52}$ is the neutrino luminosity in units of $10^{52}$ erg s$^{-1}$, $\rho_{14}$ is the local density in units of $10^{14}$ gm cm$^{-3}$, $f_{con}$ is the fraction of energy flux transported by convection (moderately less than unity for turbulent convection), and $\ell_p = p/\rho g$ is the pressure scale height or mixing length. The flow is in the MHD limit because a hot nuclear fluid is an excellent electrical conductor, with current carried by degenerate relativistic electrons.

The magnetic Reynolds number is $\mathcal{R}_m \sim 10^{17}$ (4), as compared to $\mathcal{R}_m \sim 10^{10}$ in the Sun.

Young neutron stars are also rapid rotators, as indicated by pulsar period measurements. Just after collapse, a neutron star experiences strong differential rotation even if its progenitor rotated as a solid body, because the neutron star equation of state is harder than the equation of state of the pre-collapse object, and because the angular momenta of mass shells are conserved, to an excellent approximation, during collapse (TD93a). This suggests that an $\alpha$- $\Omega$ dynamo may operate in a nascent neutron star if its initial spin period is short enough.

Conventional formulations of mean-field dynamo theory are flawed at the foundations (5). We have conjectured that in turbulent MHD flows, most of the magnetic energy becomes concentrated in thin flux ropes when the field pressure exceeds the turbulent pressure at the Kolmogorov scale (4,6). Present 3-D MHD simulations do not command sufficient computational power to resolve and follow such structures under realistic astrophysical conditions (6). It is likely that a valid mean-field theory for fast dynamos can be formulated using such flux fibrils as "basis states," since reconnection occurs rapidly at the discrete sites where fibrils cross and overlap. The macroscopic mean field $\langle \mathbf{B} \rangle$ would then be a spatial average over arrays of flux fibrils.

Such a self-consistent fast dynamo theory would share many features with conventional mean-field theory. In particular, it would exhibit a threshold criterion for the growth of the lowest-order magnetic multipoles, quantifying the competition between field amplification (due to helical convection + differential rotation) and the disorganizing influence of turbulent diffusion. In a star, this criterion can be expressed as a critical value of the Rossby number, defined as $Ro = P_{\text{rot}} / \tau_{\text{con}}$, where $P_{\text{rot}}$ is the rotation period. An $\alpha$- $\Omega$ dynamo succeeds in the limit $Ro \ll 1$, with the precise threshold value $[Ro]_{\text{crit}}$ depending upon details of the hydrodynamic flow, such as the depth of the convection zone and the distribution of differential rotation. Studies of magnetic activity in rotating late type main sequence stars strongly suggest that $[Ro]_{\text{crit}} \approx 1$ in these stars [e.g. (7)].

Based on these (observational and theoretical) ideas, we have conjectured (8) that $Ro$ is also a key parameter for neutron star dynamos, with fast-rotating ($P_{\text{rot}} \lesssim \tau_{\text{con}} \sim 3$ ms) young neutron stars supporting efficient $\alpha$- $\Omega$ dynamos. Such a threshold effect would naturally give rise to a bimodal population of neutron stars. The dipole fields of the stars born as rapid rotators ("magnetars") would greatly exceed those of ordinary pulsars, which are believed to be born with $P_{\text{rot}} \geq 10$ ms ($Ro \gg 1$), based on extrapolation of the Crab pulsar period (9) and pulsar population studies (10). Very strong fields are possible, since the dynamical saturation field for neutron star convection is $B_{\text{sat}} = (4\pi \rho)^{1/2} V_{\text{con}} \sim 10^{16}$ G, and toroidal fields as large as $B_\phi \sim 3 \times 10^{17} (P_{\text{rot}} / 1$ ms$)^{-1} \text{G}$ would be generated if the free energy of differential rotation was converted to magnetic energy. Scaling arguments (8) suggest that the (initially) rapid rotators would acquire dipole fields $B \sim 10^{14}$-$10^{15}$
G, about $10^2$ times stronger than pulsar dipole fields.

At $t \sim 30$ s after collapse, a neutron star is cool enough that neutrinos escape directly from its interior. The onset of strong stable stratification in the star at this time helps trap the entrained magnetic field, in principle up to strengths $B \sim 10^{17}$ Gauss (11,4). This occurs before the formation of the neutron star crust, thus the strength of the dipole field is not limited in any fundamental way by the tensile strength of the crust (4).

This completes our discussion of the basic magnetar conjecture. We now mention a potentially important elaboration. The minimum rotation period of a stable neutron star, $P_{\text{crit}} \sim 1$ ms, lies not too far below the $\alpha-\Omega$ dynamo threshold period of $P_{\alpha \Omega} \sim 3$ ms. [Note that accretion-induced collapse of a white dwarf to a neutron star with $P \sim 1$–3 ms will occur only if the dipole field of the dwarf is stronger than $\sim 10^7$ G, to allow a long enough spin period in accretion equilibrium (14).] Objects with larger angular momenta undergo a rotationally-supported bounce at sub-nuclear densities, and must shed angular momentum to become true neutron stars. Such “fizzlers” have an enormous amount of free energy in differential rotation which will stretch the toroidal field $B_\phi$; but the absence of a strong bounce shock and their increased neutrino transparency may keep them cold enough to prevent a convective instability (14). The evolution of these objects remains a challenging area for study. One possible outcome for strong fizzlers is that they produce a subset of the observed weak-field millisecond pulsars, as has often been suggested (14). In any case, we emphasize that magnetars might form only within a specific range of neutron star angular momenta, bounded both above and below by two distinct physical thresholds.

Another conceivable mechanism for magnetar formation is simple magnetic flux conservation in the accretion-induced collapse of a very strongly magnetized white dwarf [e.g. (15)] or in the collapse of a strongly magnetized core of a massive star [e.g. (4)]. Indeed, two white dwarfs with fields approaching $\sim 10^9$ G are known (16), although these stars are isolated rather than accreting. This scenario requires: (a) that the magnetic flux is generated in an earlier phase, presumably a convective episode within the progenitor star of the white dwarf or supernova; and (b) that the dipole field is not affected by turbulence and differential rotation during and after the collapse. It is not clear that these conditions are satisfied in nature. Post-collapse convection has a larger ratio of kinetic energy to gravitational binding energy than any previous phase of convection driven by nuclear burning, thus it is capable of producing the largest magnetic flux in dynamical saturation [§8 in (4)]. Furthermore, convective carbon burning, the last convective phase in a white dwarf progenitor, is only marginally capable of generating $\sim 1 \times 10^9$ G in a white dwarf (close to the maximum field observed), which translates to a neutron star with $B \sim 1 \times 10^{14}$ G assuming flux-freezing, almost an order of magnitude lower than the dipole field inferred for SGR 0526–66 (12). Earlier convective phases could produce only weaker fields. In sum, the assumption of a conserved dipole magnetic flux is dubious in a newborn neutron star which
is undergoing strong mixing and differential rotation.

II. OBSERVABLE CONSEQUENCES: SGRs, AXPs (& GRBs?)

Magnetars spin down too rapidly to be easily detectable as radio pulsars. The exterior dipole field energy—which is probably a small fraction of the total magnetic energy—exceeds the rotational energy after only $\sim 200 (B_{\text{dipole}}/3 \times 10^{14} \text{ G})^{-4}$ yrs. Thus magnetism rapidly becomes the dominant free energy source within isolated magnetars. As their fields evolve via diffusive processes (crustal Hall drift, interior ambipolar diffusion) catastrophic releases of magnetic energy plausibly occur, roughly analogous to stellar flares (8). In particular, crustal fractures driven by magnetic stresses will inject an Alfven pulse into the magnetosphere, leading to the formation of a trapped pair plasma. Cooling of this plasma in a very strong magnetic field provides a promising mechanism for the ultra-luminous soft gamma repeater (SGR) bursts (12). Several independent arguments (12) point to a dipole field of $\sim 6 \times 10^{14} \text{ G}$ in SGR 0526-66 (the source of the March 5, 1979 burst).

Fractional heating by the diffusing magnetic field keeps the core of a young magnetar relatively hot (13, 12). This heating is balanced by modified URCA cooling at a temperature $T \simeq 2.4 \times 10^6 (B/10^2 B_{\text{QED}})^2 (\rho/7 \times 10^{14} \text{ g cm}^{-3})^{-1}$ K (4). One observational consequence is an anomalously large thermal surface X-ray luminosity, $L_X \simeq 6 \times 10^{34} (\rho/7 \times 10^{14} \text{ g cm}^{-3})^{0.1} (t/10^4 \text{ yr})^{-0.3} \text{ erg s}^{-1}$ if the magnetic field is just beginning to decay at age $t$. Neutron superfluidity does not change the $T - B$ relation to first order, but does decrease the decay rate of the magnetic field, thereby allowing a stronger field and higher (surface) temperature at a fixed age. A magnetic field in the range $\theta_{\text{max}}^{1/2} \lesssim B < B_\mu \equiv (4\pi \mu)^{1/2} \sim 6 \times 10^{15} \text{ G}$ (where $\mu$ is the shear modulus of the crust and $\theta_{\text{max}} \sim 10^{-4} - 10^{-2}$ is the limiting strain) induces multiple small scale fractures as it is dragged through the crust by ambipolar diffusive motions in the core. The resulting seismic waves convert to a continuous stream of low-amplitude Alfven waves in the magnetosphere, and provide a possible mechanism for energizing the non-thermal radio pulsed power around SGR 1806-20 (12).

Another class of magnetar candidates are the “anomalous X-ray pulsars” (AXPs) listed in Table 1. These sources have soft-spectrum X-ray emissions modulated on spin periods $\sim 10$ s, with histories of uniform, steady spin-down. They have no detected companions or orbital pulse modulations, but at least two AXPs are associated with young supernova remnants (SNRs). Many of these properties are shared by SGRs.

In the magnetar model we can estimate the dipole fields of AXPs in two ways. We can either ask what field is required to drive the present, measured spin-down rate $[B_{\text{dipole}}(P, P)]$, in column 4 of Table 1), or how strong the field must have been to spin down the star to period $P$ (from a much smaller initial period) in the age of the associated supernova remnant (SNR) $[B_{\text{dipole}}(P, t_{\text{SNR}})]$, in column 5 of Table 1). In both cases the spin-down mechanism is idealized as vacuum magnetic dipole radiation. Note that for
### TABLE 1. The Anomalous X-Ray Pulsars (AXPs)

<table>
<thead>
<tr>
<th>Object (references)</th>
<th>$P$</th>
<th>$\dot{P}$</th>
<th>$B_{\text{dipole}}(P, \dot{P})$</th>
<th>$B_{\text{dipole}}(P, t_{\text{SNR}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E 2259+586 (17,18)</td>
<td>6.98 s</td>
<td>$5.0 \times 10^{-13}$</td>
<td>$0.7 \times 10^{14}$ G</td>
<td>$3 \times 10^{14}$ G $^a$</td>
</tr>
<tr>
<td>1E 1048–5937 (19,20)</td>
<td>6.44 s</td>
<td>$1.5 \times 10^{-11}$</td>
<td>$4 \times 10^{14}$ G</td>
<td>$^b$</td>
</tr>
<tr>
<td>4U 0142+614 (24,22)</td>
<td>8.69 s</td>
<td>$2.3 \times 10^{-12}$</td>
<td>$2 \times 10^{14}$ G</td>
<td>$^b$</td>
</tr>
<tr>
<td>RXJ 1838–0301 (23)</td>
<td>5.45 s</td>
<td>not yet measured</td>
<td>—</td>
<td>$2 \times 10^{14}$ G $^c$</td>
</tr>
</tbody>
</table>

$^a$Age of associated SNR $t_{\text{SNR}} \sim 1.3 \times 10^4$ yrs (18).

$^b$No associated SNR yet identified.

$^c$Age of associated SNR $t_{\text{SNR}} \sim 3 \times 10^4$ yrs, roughly estimated (23).

1E2259+586, the two determinations of $B_{\text{dipole}}$ are discrepant by a factor $\sim 4$. This might indicate a decrease in the dipole field strength during the star's lifetime, perhaps by the action of the Flowers-Ruderman instability [ (27); §15.2 in (4)], or a systematic error in the age of the SNR.

In this model, a number of possible mechanisms exist for imparting a large proper motion to AXPs at birth (8). A large recoil has been inferred for SGR 0526-66 (8,24,25). From the X-ray and radio observations of ref. (26), interpreted as in ref. (18), we infer that 1E2259+586 is displaced 3.1' to the east of the center of its associated SNR; this implies a transverse velocity $V_{\text{trans}} \simeq 340 \,(D/5\text{kpc}) \, \text{km} \, \text{s}^{-1}$. The recoil of RXJ 1836–0301 is more difficult to estimate accurately because of the irregular shape of the remnant; however it is clear that the star is significantly displaced, by $\sim 20'$, to the SE of the centroid of the X-ray emissions. This gives a rough first estimate $V_{\text{trans}} \sim 600 \,(D/3\text{kpc}) \, \text{km} \, \text{s}^{-1}$, using the SNR age estimate quoted in Table 1. These velocities are large enough to unbind the neutron stars from low-mass companions. Another reason to doubt that AXPs are accreting low-mass X-ray binaries (LMXBs) is based on the paucity of these sources in the Galaxy. Such X-ray binaries should persist for $\sim 10^7$–$10^8$ yrs (the gravitational wave decay time of the orbit) so if one sees a few of these sources at age $10^4$ yrs, thousands of older ones should be observed. Note that these arguments against the LMXB interpretation do not preclude the possibility that AXPs are accreting from fossil disks (17,28).

A possible connection between the SGRs and the non-thermal (classic) GRBs is suggested by the relatively hard March 5, 1979 burst. If all magnetars acquire large recoil velocities like SGR 0526-66, and if they remain magnetically active at an age of $\sim 10^8$ yr, then they are a possible source of GRB's in the halo (8). Moderate beaming of the gamma-ray emissions allows a nearly isotropic distribution of bursts, which—in a range of the relevant parameter space—can fit data from BATSE and PVO (29,30). A related model involves weakly-bound magnetars orbiting in the non-spherical galactic potential (31). In some respects, magnetic reconnection is a theoretically advantageous energy source for halo gamma-ray burst sources (8), since gravitational and nu-
clear energy releases occur in bulk baryonic matter, which has many degrees of freedom into which energy can be thermalized and degraded via adiabatic expansion. Magnetic flares on neutron stars would be relatively free of baryonic pollutants, thus could more plausibly generate the observed hard spectra of GRBs. In addition, magnetic stellar flares are known to exhibit chaotic variability over a wide range of time scales, as observed in GRBs. The main disadvantage to powering halo GRBs with strong magnetic fields is that the bursting activity should be largest when most magnetic dissipation occurs, namely at young ages when the neutron stars still reside in the galactic disk.

In general, galactic halo models require special conditions to fit the observed angular and flux distributions of GRBs. Cosmological GRB models explain these observations more generically. One of us (32) has noted that a naked proto-magnetar (which can form in the accretion-induced collapse of a white dwarf) or a magnetized neutron disk (which forms in the merger of massive double-degenerate binary) will generically release a Poynting flux dominated MHD wind of the required luminosity \( L \approx 10^{50} \text{ erg s}^{-1} \). Reconnection in the wind at moderate to low scattering depth excites Alfvén turbulence that Compton upscatters the entrained photons, reproducing the power law spectral indices and breaks energies observed in classical GRBs in a straightforward manner.

This work was supported by the Texas Advanced Research Project, Grant No. ARP-279, and by the NSERC of Canada.

**APPENDIX** We now explain how steady-state neutrino diffusion can produce a negative entropy gradient, driving convection beneath the neutrinosphere in an isolated, hot neutron star (3,4).

The transport equation implies a radiative temperature gradient \( (dT/dr)_{rad} \propto -F_{\nu} \rho T^{\alpha-3} \), where \( F_{\nu} \) is the neutrino flux, \( \rho \) is density, and the Rosseland mean neutrino opacity is assumed to vary as \( \kappa_{\nu} \propto T^{\alpha} \). This opacity is dominated by nucleon scattering and beta processes: \( \nu n \rightarrow \nu n \), \( \nu p \rightarrow \nu p \), and \( \nu_e n \rightarrow e^- p \), which have cross sections varying with neutrino energy as \( \sigma \propto E_{\nu}^2 \), thus \( \alpha = 2 \) in the nondegenerate layers just beneath the neutrinosphere. [This steepens to \( \alpha > 2 \) at the onset of degeneracy (3) which exacerbates the convective instability.] From the equation of hydrostatic equilibrium, one finds \( (dT/dP)_{rad} \propto (F_{\nu}/g) T^{-1} \) for \( \alpha = 2 \). Thus the run of temperature with pressure in the outer layers (where \( F_{\nu} \) and \( g \) are roughly constant) is \( T_{rad} \propto P^{1/2} \). In hot nuclear matter with adiabatic exponent \( 1.4 < \Gamma < 1.5 \) (33) the adiabatic profile, on the other hand, is \( T_{ad} \propto P^{(\Gamma-1)/\Gamma} = P^{0.28-0.33} \) (cf. (34)). The radiative gradient is superadiabatic, and entropy-driven convection ensues.

Within a Type II supernova, residual accretion heating at the stellar surface could flatten the entropy profile of a proto-neutron star and temporarily suppress internal convection [e.g., (35)], but this does not happen in neutron stars formed via accretion-induced collapses of white dwarfs.
REFERENCES

1. A. Burrows, this volume
30. H. Li and R. Duncan, this volume.

117